Telecommunications Engineering

Universidad Carlos III de Madrid

Statistics

Assignment 4: Random Vectors & Stochastic Processes

| **Group** | **Students** | **Signatures** |
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**IMPORTANT:** *The teachers of this course apply a ‘zero tolerance’ policy regarding academic dishonesty. Students that sign up this document agree to deliver an original work. The breach of this commitment will result in academic punishment.*

**Observations:**

Solve the exercises in the **Assignment4.pdf** file. **Note:** It is advisable to consult the manual for basic operation of MATLAB / Octave available on the website of the course.

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Available on GitHub: <https://github.com/alonso-herreros/uni-stat-lab4>

# 1. Random Vectors

## 1.1. Exercise 1.

#### Choose a point randomly in the interval and a second point also randomly in the interval . Define the r.v. as “the position of the first point” and the r.v. as “the distance of the second point respect to the first”.

#### Determine, theoretically, the joint density function of (, ).

Let and

Now, let

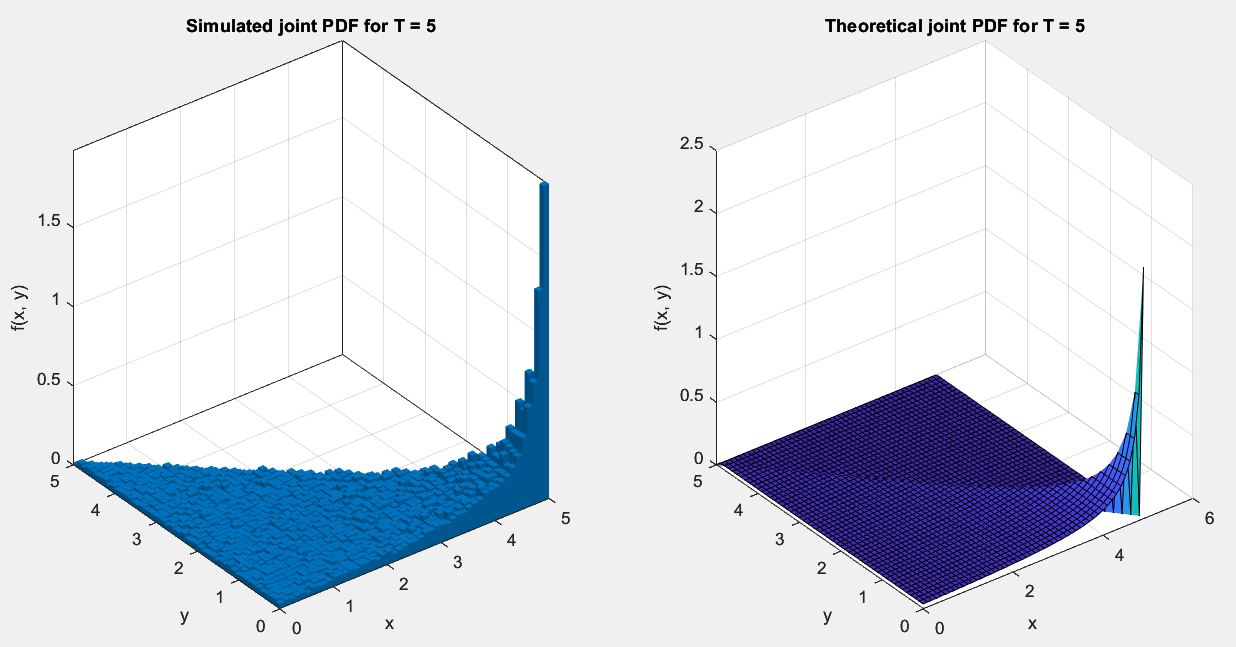
For , we have

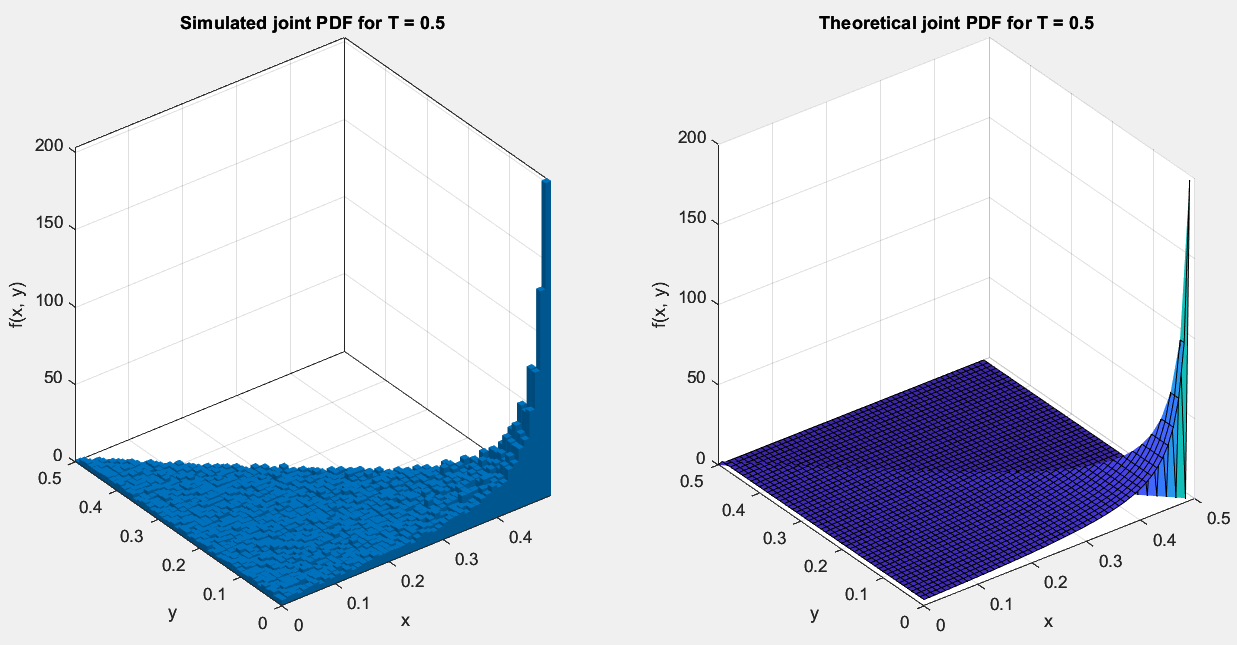
We may conceptually use

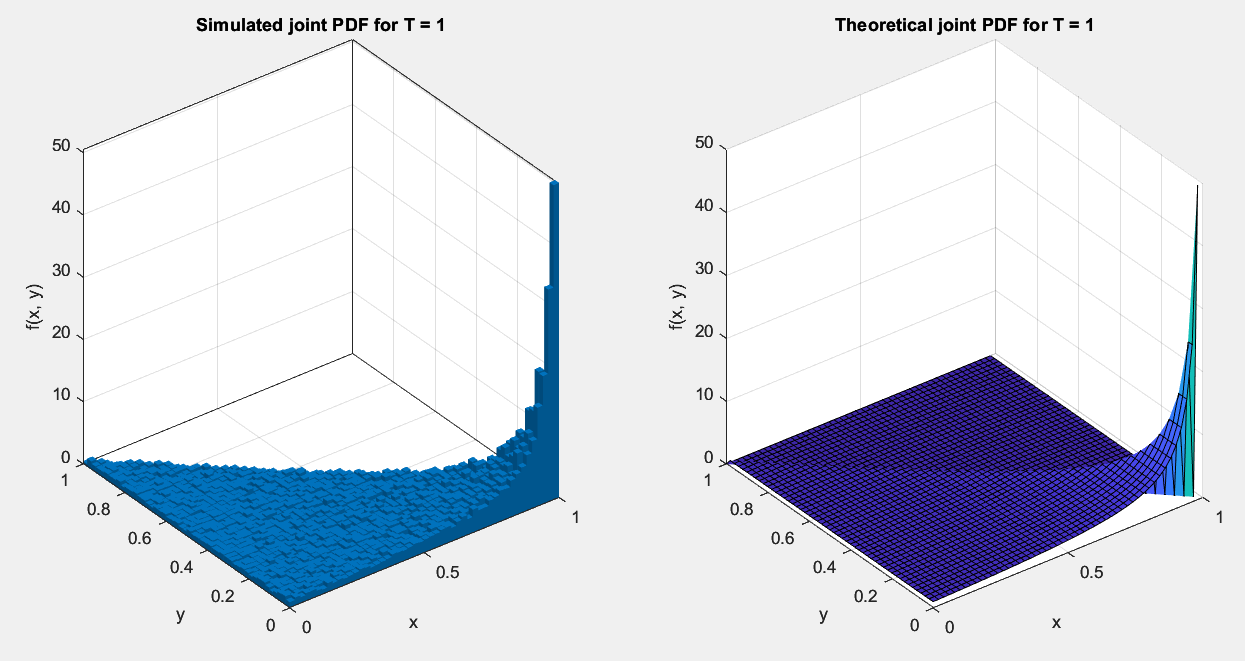
Since these are uniform distributions, we may use the formula , and the analogous for , while setting the limits x and y.

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This formula was tested by plotting it, along with the graph generated from a simulation of this process, and visually comparing both results, with different values of T.

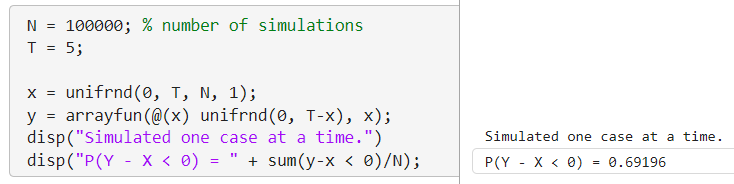






#### Compute by simulation using MATLAB/Octave, P (Y − X < 0) for T = 5.

The simulation is quite simple. First, a vector x is created with random values from the variable , according to a uniform distribution between 0 and T, using the function unifrnd. Then, the vector x is created by generating a random value between 0 and T-x for each entry in the vector x, using unifrnd and arrayfun. The intermediate step of generating a random point between x and T was not necessary, and since this is a very simple process, it was skipped.



## 1.2. Exercise 2.

#### Let X and Y be independent r.v.’s with X continuous and Y discrete, given by X ∼ U (−1, 1) and

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#### where the notation indicates that the r.v. Y takes value 0 with probability 1/4 and value 1 with probability 3/4. Considering :

#### Determine .

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We have , so we may think of three ranges:



With all of this:

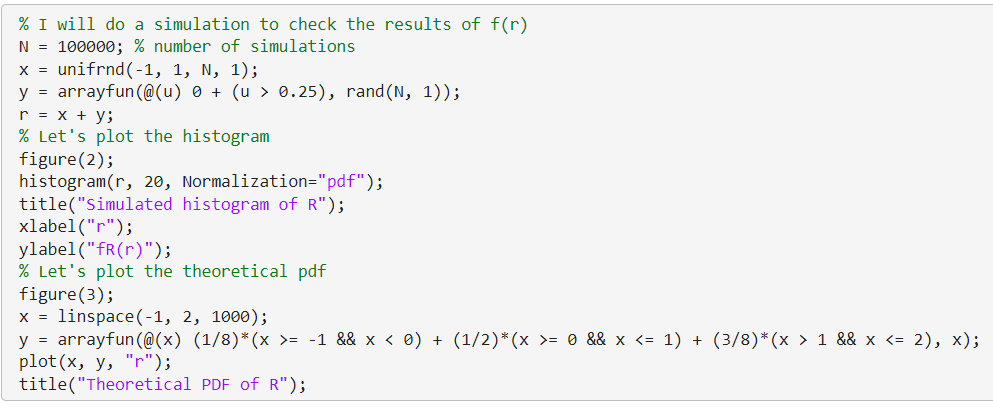
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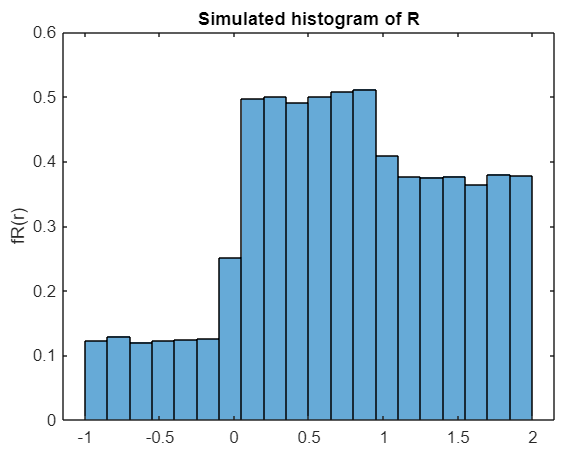
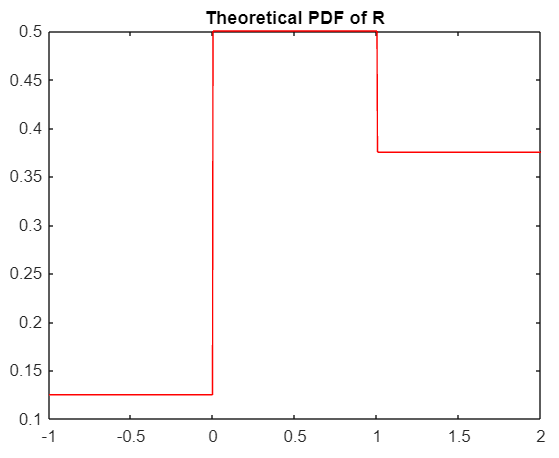
#### Calculate theoretically E [R] and V ar [R].

#### Check with MATLAB/Octave the results obtained in a) and b)

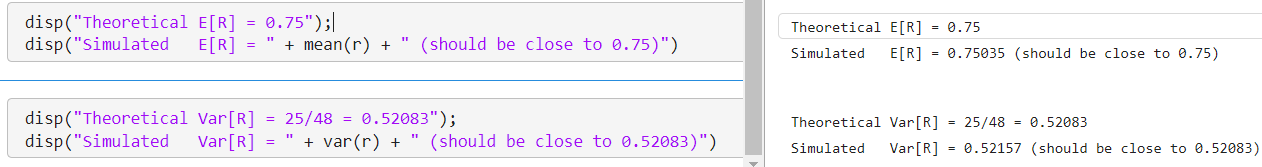
This was done directly when calculating the theoretical formulas, to check their validity. The code is quite simple, and self explanatory at this point.

For part a), the process was simulated and the theoretical function was plotted side by side with a PDF-normalized histogram of the simulation to check if the values matched.





The graph with the simulated data is quite consistent with the theoretical values.

The same simulation data that was created to check the result in part a) was used to check the result in part b).  


# 2. Stochastic Processes

## 2.1. Random Walk

#### A random walk is a discrete stochastic process , given by the sum of n i.i.d. random variables, i.e.,

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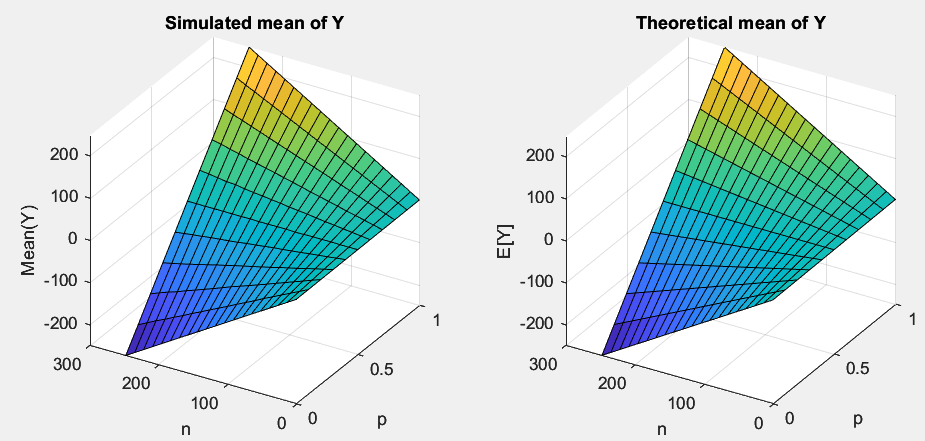
#### An example of random walk is such that where takes value with probability and value with probability .

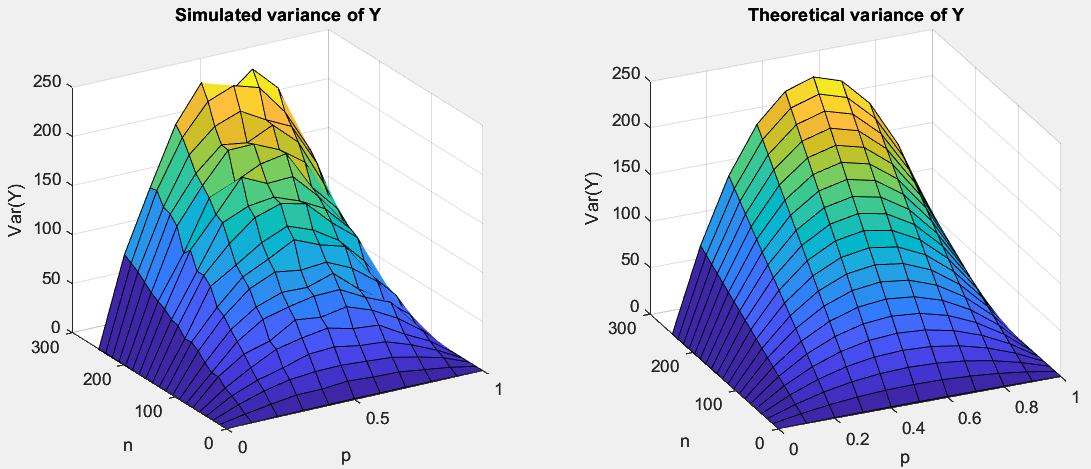
#### Determine analytically the mean and the variance of the process . Are the mean and the variance of the stochastic process constant when p = 1/4?

When , we have the following expected value and variance:

Therefore, for , the variance and expected value are not constant, since they depend on .

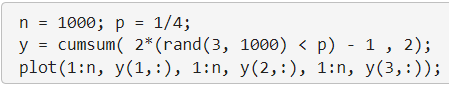
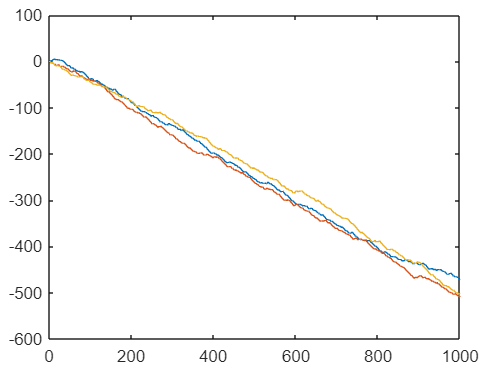
These theoretical formulas were plotted against simulated data to check their validity, using MATLAB.





The plots show enough similarity to conclude that the theoretical formulas are accurate. The code wasn’t required for this report, but it can be found with the rest of the exercises in the GitHub repository linked at the top of this document.

#### Generate with MATLAB/Octave three realizations (overlapping the graphs) of a Bernoulli process with p = 1/4 and n = 1000. Is the obtained result consistent with a)?

The process was simulated using a very simple piece of MATLAB code:  
 

The resulting plot was also consistent with the theoretical expected value.

## 2.2. Harmonic Processes

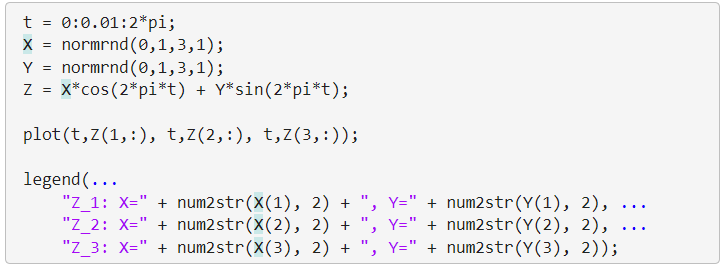
#### Let X and Y be two independent r.v.’s normally distributed with parameters and . Define the harmonic process as:

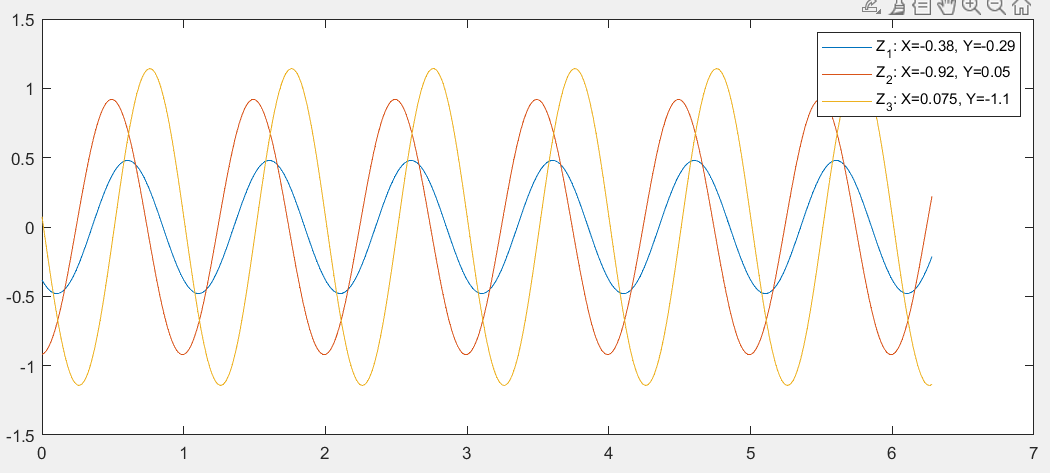
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#### Generate with MATLAB/Octave three realizations (overlapping the graphs) of the process with

The first time tackling this problem, X and Y were assigned random values for each point in time, leading to a plot full of lines everywhere, from which no data could be obtained. This first plot, which made no sense, was an accurate representation of the process given to the program, but it was a wrong interpretation of the question.

The following simple MATLAB code was used to generate a plot with the three realizations of the stochastic process.





## 2.3. Harmonic process with white noise

#### Consider the stochastic process of sinusoid type with random phase and white noise defined by:

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#### where , , and independent from .

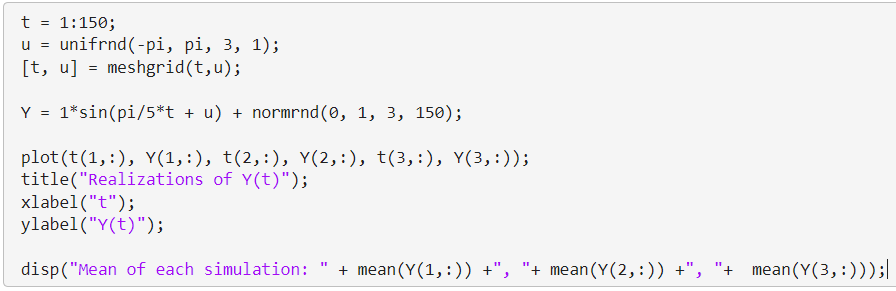
#### Determine analytically the mean of the stochastic process Y (t). Is the mean of the process Y (t) constant?

Yes, the mean is constant and equal to , independently of , as we can see from the calculations. The noise-inducing term has a mean of 0, so it doesn’t affect the mean here. The random variable introduces a random phase, ranging from to . Since it covers all possible phases, and it follows a uniform distribution, the output phase is unpredictable, and the average becomes .

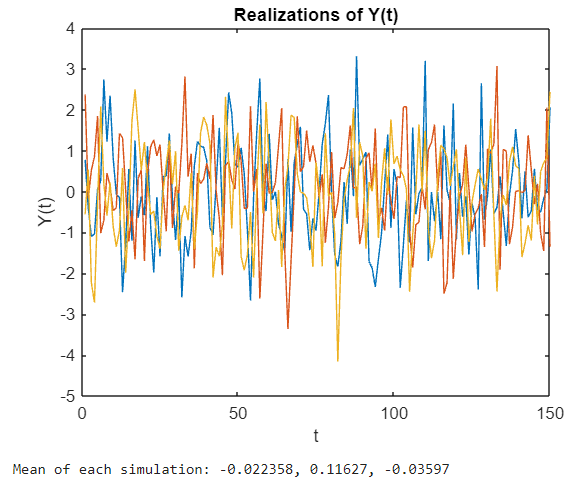
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#### Generate with MATLAB/Octave three realizations (overlapping the graphs) of the process Y (t) with t = 1 : 150. Is the obtained result consistent with a)?

Using the given parameters, the MATLAB code used was the following:



And the output was as shown here:



The simulated means are close to 0, so the theoretical and simulated values are consistent with each other. While the plots seem to oscillate around 0, due to the large amount of noise and the high frequency, it’s very hard to obtain any significant information from the graph.

By lowering the variance of the noise-inducing term and lowering the frequency , more understandable plots can be generated. The plot below contains three realizations of this harmonic process with white noise, except the variance of was changed to 0.2 and the frequency was changed to . The signals are noisy and have a random phase, but their behavior is easier to see.

